Ma/CS 6c
Assignment #5
Due Wednesday, May 7 at 1:00 pm.

IN ALL PROBLEMS BELOW YOU CANNOT USE THEOREM 1.11.5 IN THE NOTES (since the exercises below form parts of the proof of that theorem). ALSO "FORMULA" OR "WFF" MEANS "WELL-FORMED FORMULA IN PROPOSITIONAL LOGIC BUILT USING PROPOSITIONAL VARIABLES, PARENTHESES, AND ONLY $\neg, \Rightarrow$."

(40%) **1.** (i) Show that for any formula A,

$$\vdash (\neg\neg A \Rightarrow A).$$

(ii) Prove Proposition 1.11.8 in the notes:

Let S be any set of wffs and A any wff. If $S \cup \{A\}$ is formally inconsistent, then $S \vdash \neg A$.

(iii) Show that if S is formally inconsistent, then $S \vdash A$ for any wff A.

(30%) **2.** Prove Proposition 1.11.9 in the notes:

Let S be any set of wffs and A, B any wffs. Then:

$$S \cup \{A\} \vdash \neg B \text{ iff } S \cup \{B\} \vdash \neg A.$$

(30%) **3.** (i) Show that
$$\neg(A \Rightarrow B) \vdash A$$
and
$$\neg(A \Rightarrow B) \vdash \neg B$$

(ii) Prove Lemma 1.11.12 in the notes:

If $\bar{S}$ is a formally consistent and complete set of formulas and ν is the valuation defined by
$$\nu(p_i) = \begin{cases} 1, & \text{if } p_i \in \bar{S} \\ 0, & \text{if } p_i \notin \bar{S}, \end{cases}$$
then for any formula A,
$$\nu(A) = 1 \text{ iff } A \in \bar{S}.$$