Ma/CS 6c

Assignment #3

Due Wednesday, April 23 at 1:00 pm.

(35%) **1.** (i) Prove that $\{\Leftrightarrow, \wedge\}$, $\{\Leftrightarrow, \vee\}$ are *not* complete.

(ii) Prove that $|, \downarrow$ are the *only* complete binary connectives.

(35%) **2.** Let G be a graph with set of vertices V. A *coloring* of G with k colors ($k = 1, 2, \dots$) is a map $c : V \to \{1, 2, \dots, k\}$ so that if $x, y \in V$ are adjacent, then $c(x) \neq c(y)$. By a *finite subgraph* of G we mean a graph consisting of finitely many vertices of G, with two such vertices adjacent iff they are adjacent in G.

Assume that G is a graph with infinitely many vertices $V = \{x_1, x_2, x_3, \dots\}$. Show that if every finite subgraph of G has a coloring with k colors, then G has a coloring with k colors.

(30%) **3.** Let $\{A_1, A_2, A_3, \dots\}$ be an infinite set of formulas in propositional logic. Assume that for every valuation v there is some n (depending on v) such that $v(A_n) = 1$. Show then that there is some fixed m with

$$A_1 \vee A_2 \vee \cdots \vee A_m$$

a tautology.